

Assignment Report

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Department:

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Course:

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Assignment:

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| *Based upon:* |  | Click or tap here to enter text. |

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## discretization

The differential form of the 1D steady-state convection-diffusion transport equation,

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

can be discretized using the finite volume method by evaluating within cells along the -axis. For a cell the left neighbouring cell is denoted and the right is . The face between cell and is , and is between and . Evaluating (1) for any cell , equation (1) is integrated between the cell face locations, to . If is constant, the integration gives,

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

### Non-boundary cells

Starting with the convective term on the LHS of (2), the face values, and , must be described as a function of the neighbouring cell centre values. Using a 2nd order central difference scheme (CDS), and , can be expressed as,

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

If instead using the 1st-order upwind difference scheme (UDS) they can be expressed as,

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

For the diffusion term on the RHS of (2) the differentials can also be expressed using the central difference scheme, giving,

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Substituting (5) and (3) into (2) gives,

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

If using (4) instead of (3) the substitution gives,

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

Realizing that the terms within the parentheses are constants (6) and (7) can be expressed for the ’th cell as,

|  |  |  |
| --- | --- | --- |
|  | where: | (8) |

for .

### Boundary cells

Using Dirichlet boundary conditions, (3) and (4) must be modified for the boundary cells to,

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

where and are known values. For the case of CDS the boundary values are exact, so no truncation error arises here, but for UDS it is still 1st order accurate at one of the boundaries but exact at the other.

Equation (5) must also be modified with a one-sided scheme. In this case a 1st order is used,

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

Substituting (9) and (10) into (2) gives (11) for cell

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

and (12) for cell ,

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

These doesn’t fit into the first equation in (8) but the definitions therein can be substituted into (11) and (12) to give (13) for cell ,

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

and (14) for cell ,

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

### Solving the system

An equation for every cell is now defined with equation (8), (13), and (14). These can be combined to give the linear system,

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

where is a tridiagonal matrix,

## Solution computed

## Order of discretization error